Econ 110/PoliSci 135
Section 11 Notes

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1 Announcements

The final will emphasize topics covered in the second half of the class. Here is a list of topics we covered post-midterm:

1. Bargaining (with complete information, with and without discounting)
2. Repeated Games
   - Infinite
   - Finite
3. Games of Asymmetric Information (bargaining)
4. Principal-Agent Models
   - Adverse Selection
   - Moral Hazard
5. Signaling Games

2 Perfect Bayesian Equilibrium

In order to solve general games of imperfect information, we need more than just the strategies players take. We need to also specify beliefs that players have about where they are in the tree. These beliefs may be necessary to say whether an action is optimal. The key issue is that an off the equilibrium path information set of a player requires beliefs about how that information set was reached in order for that player to make a decision.[1]

3 Signaling Games

3.1 Overview

Signaling games, are games of asymmetric information where there are different types of senders and the receiver does not know which sender she faces. The sender has a costly signal, and from this signal, the receiver may or may not be able to draw some inference about the sender's type. The receiver must take an action that affects both the sender and the receiver.

[1] Material in this handout borrows from Matt Levy and Josh Tasoff's 2007 notes

3.2 Example: Beer and Quiche

In one of the canonical treatments of the signaling game, we consider two gentlemen having a dispute over honor. Lord Signaler has offended the honor of Baron von Receiver, and the Baron is on his way to challenge Lord Signaler to a duel this morning. If Lord Signaler is tough, he will win the duel. If he is wimpy, he will lose. However, either way, Lord Signaler would prefer Baron von Receiver to rescind his challenge.

Lord Signaler knows his own type, but Baron von Receiver only knows that there is a .1 probability of weakness and a .9 probability of strength.

Sitting down to his breakfast, Lord Signaler sees the Baron coming in the distance. His Lordship must still, however, choose what to eat for breakfast. If he is weak, he would gain an extra 2 utility from quiche and 0 from a beer. Conversely, a tough Lord Signaler would get an extra 0 utility from a quiche and 1 utility from a beer.

3.2.1 Separating Equilibria

First we want to find all the separating equilibria. In a separating equilibrium, the two types of Lord Signalers take different actions. Thus there are two candidate separating equilibria to consider. One in which wimpy eats quiche and tough drinks beer, and one in which wimpy drinks beer and tough eats quiche.

Let's consider the first, more intuitive separating profile. Lord Signaler's strategy is eat quiche if wimpy, and drink beer if tough. Now we must specify Baron von Receiver's strategy and beliefs contingent upon seeing Lord Signaler's breakfast. From these actions Baron von Receiver can perfectly infer Lord Signaler's type. Thus Baron von Receiver will believe that he is facing a tough guy with probability one when he sees beer and believe that he is facing a wimpy guy with probability one when he sees quiche.

Given these beliefs we must now specify a best response for the Baron. The Baron will choose to not duel given he sees beer because $0 > -1$, and the Baron will choose to duel if he sees quiche because $1 > 0$. Thus our candidate strategy and beliefs are given by:

- Lord Signaler
  - If tough, beer.
  - If wimpy, quiche.
- Baron von Receiver
  - If observes beer, believes $prob(tough \mid beer) = 1$, and chooses *Don't*.
  - If observes quiche, believes $prob(tough \mid quiche) = 0$, and chooses *Duel*.

This is a best response for the Baron by construction. Now we have to go back and check to make sure that neither a wimpy nor tough Lord Signaler would have any incentive to deviate. The utility of the tough signaler is 3. If the tough signaler deviates to eating quiche, he would get 0, so the tough guy has no incentive to deviate. The utility of the wimpy signaler is 1. If the wimpy signaler deviates to drinking beer, he would get 1 as well. Thus the wimpy signaler has no incentive to deviate. Therefore the strategy above is a Perfect Bayesian Equilibrium.

Now let us consider the other separating equilibrium:

- Lord Signaler

- If tough, quiche
- If wimpy, beer

- Baron von Receiver
  - If observes beer, believes $prob(tough \mid beer) = 0$, and chooses *Duel*.
  - If observes quiche, believes $prob(tough \mid quiche) = 1$, and chooses *Don't*.

Under these strategies the tough guy is getting 2 and would get only 1 by drinking beer. However the wimpy guy is getting -1 and can profitably deviate to 3 by eating quiche. Thus this is not a PBE.

3.2.2 Pooling Equilibria

Pooling equilibria occur when both types take the same action. In this game, tough and wimpy can pool on quiche, or tough and wimpy can pool on beer. First let's see if we can construct an equilibrium where they both pool on quiche. If both are pooling on quiche, then the Baron's expected utility from dueling after seeing quiche is $.9(-1) + .1(1) = -.8$, whereas his expected utility from don't dueling is 0. Therefore the baron will "don't duel" after seeing quiche. In order to prevent deviations out of the pooling, we have to make beer a bad option for Lord Signaler. Thus after seeing beer let's assume that Baron von Receiver duels. We need to construct some beliefs that legitimize dueling. If x is the probability of a tough guy drinking beer, then $EU_R(duel|beer) = -x+(1-x) \geq 0 = EU_R(don't|beer) \Rightarrow x \leq \frac{1}{2}$. Thus for all $x \leq \frac{1}{2}$, the candidate pooling on quiche PBE are given by:

- Lord Signaler
  - If tough, quiche.
  - If wimpy, quiche.

- Baron von Receiver
  - If observes quiche, believes $prob(tough \mid quiche) = .9$, and chooses *Don't*.
  - If observes beer, believes $prob(tough \mid beer) = x$, and chooses *Duel*.

We still have to check to make sure that neither tough nor wimpy Lord Signaler have any incentive to deviate. Tough is getting 2 and could only get 1 if he deviates by drinking beer. Wimpy is getting 3 and could only get -1 if he deviates by drinking beer. The infinite profiles above are PBE.

Check on your own to show that there also exists an infinite number of pooling equilibria where both pool on beer. Characterize all the PBE. What are the beliefs that Baron von Receiver must have for this to be possible?

Here is one pooling PBE on beer. Check on your own that no player has a profitable deviation.

- Lord Signaler
  - If tough, beer.
  - If wimpy, beer.

- Baron von Receiver

– If observes beer, believes $prob(tough \mid beer) = .9$, and chooses *Don't*.

– If observes quiche, believes $prob(tough \mid quiche) = 0$, and chooses *Duel*.

4 The Intuitive Criterion

4.1 Overview

The Intuitive Criterion is a restriction on the *beliefs* that we let a receiver hold in a PBE. The basic idea is that letting them form whatever beliefs they want when there's a signal that nobody sends is just too loose. There are sometimes some types who would never send the signal, and some who are only not sending it because they think the receiver will react in a specific way. The Intuitive Criterion basically says that the receiver should never believe it's one of the former types.

4.2 Definition

Suppose we have a Perfect Bayesian Equilibrium, and that there is a signal s_0 that nobody sends.

For each type of signaler, let u_i be the maximum payoff they could get by sending s_0 such that the receiver is still responding optimally to *some* set of beliefs. Let that type's *actual* payoff in this candidate equilibrium be e_i.

The equilibrium passes the *intuitive criterion* if the receiver's beliefs in response to s_0 put a probability 0 of it being any type such that $u_i < e_i$. In other words, if a signaler of a certain type cannot possibly receive a higher payoff than the current equilibrium payoff from sending a different signal s_0, then the receiver should put zero probability weight on that type after seeing s_0.

4.3 Example: Beer and Quiche Revisited

Since all signals are sent in the separating equilibria, we need only consider the pooling equilibria. First let's check to see if the pooling equilibria on quiche survive the intuitive criterion. Remember that the intuitive criterion is a restriction on *beliefs* off the equilibrium path.

The off-equilibrium belief that we found was that $prob(tough \mid beer) = x$ where $x \leq \frac{1}{2}$. So here the receiver believes that at least half of those who deviate by choosing beer are wimps, and the rest are tough. The equilibrium utility of tough guys is 2. The best he could possibly get from choosing beer is 3. So according to the intuitive criterion, Baron von Receiver is aloud to believe that a tough guy would choose beer with some probability. The equilibrium utility of wimps is 3 and the best a wimp can get from choosing beer is 1. According to the intuitive criterion, Baron von Receiver must believe that a wimp will *never* choose beer. This implies that $x = 1$. But if $x = 1$ that contradicts our assumption that $x \leq \frac{1}{2}$.

Pooling on quiche *does not survive the intuitive criterion.* The intuition is this. If both are pooling on quiche, a tough guy could say, "Baron, check this out, I'm drinking beer. Clearly a wimp cannot possibly do any better than the status quo and so by drinking beer it must be the case that I'm a tough guy. That means you shouldn't duel me after seeing that I drink beer."

Now let's check to see if the pooling on beer equilibrium we found survives the intuitive criterion. Here the Baron's off-equilibrium belief is $prob(tough \mid quiche) = 0$. The tough guy's equilibrium payoff is 3 and so the best he can do by eating quiche is 2. By the intuitive criterion Baron von Receiver must believe that a tough guy will *never* choose quiche. This is exactly the belief we have. The wimp's equilibrium utility is 1, the best the wimp can do by eating quiche is 3 and so the intuitive criterion puts no restrictions on the probability that a wimp will eat quiche. This pooling equilibrium on beer satisfies the intuitive criterion.

Do the other pooling equilibria on beer that you found on your own survive the intuitive criterion?