Econ 110/PoliSci 135
Section 6 Notes

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1 Announcements

- The midterm will take place during lecture, Tuesday, October 22. Problem sets are due on Thursday, October 17. Solutions will be available for pick up from Evans that day. I will hold a review session probably on Sunday, Oct 20, but I will confirm the day and time later this week.
- Problem Set 2, Number 3a: Recall that to check whether a particular strategy is a Nash equilibrium, you must ask the following question for ALL the players: "If the other players play the strategy they claim, can I profitably deviate?" If no players can profitably deviate, then you have a Nash. For this question, you couldn't just draw a matrix because there are too many strategies to check.

2 Subgame Perfect Nash Equilibrium

A **subgame** is a collection of nodes and branches that satisfies three properties: 1.) It starts at a single decision node. 2.) It contains every alternative at this node. 3.) If it contains any part of an information set, then it contains all the nodes in that information set. If we're looking at a game tree, a subgame doesn't cut across any information sets.

A **Subgame Perfect Nash Equilibrium** (SGPN) is a strategy profile such that the players are playing a Nash equilibrium in each and every subgame.

A SGPN is a specific *type* of Nash Equilibrium. It can be considered a Nash equilibrium refinement, or an equilibrium concept in itself. It adds the additional assumption that players must play Nash equilibria within each subgame, but it helps us narrow down the set of solutions to more "reasonable" predictions about how the game will play out.

2.0.1 An example

Let's return to the Raider/ Target game we discussed in class. How many subgames does this game have? How many decision nodes does each player have? How many information sets does each player have?

Why is the strategy profile {Out, Cooperate; Poison Pill} a Nash Equilibrium? Is this strategy profile a SGPN? Which strategy is problematic if we want the equilibrium to be SGPN?

Now let's consider the subgame of the game. We saw from above that Poison Pill is a dominated strategy for the Target, so we can ignore it from now on. Notice that the subgame now essentially reduces to the following strategic form game:

(Let Target=Player 2 and Raider=Player 1)

	F	**C**
F	-2,-2	4,-6
C	1,2	3,1

What is the Nash equilibrium of the subgame? R will play C and T will play F. Now that we know that R plays In, he will end up at the terminal node with the payoffs (1,2). At the initial node of the game, what will R choose? What is the SGPN of the entire game?

3 Brinkmanship

3.1 Resolve

In games of brinkmanship, the fundamental issue is that there is an awful sanction that we're both so afraid of that we can't credibly threaten to bring it about. But one player can use fear about the sanction happening to exert some sort of pressure on the other player. Brinkmanship is really a game of risk taking. We play until disaster happens or until one of the players backs down because the risk has gotten too large.[1]

Three Possible Things Can Happen in games of Brinkmanship:

1. I win (you back down), there's no disaster.
2. You win (I back down), there's no disaster.
3. Disaster occurs

Resolve is a convenient summary statistic for Brinkmanship games - it tells us the maximum risk a player is willing to face before giving up. In these games, players compete by "bidding up" the risk of disaster. To calculate each player's resolve, we set the payoff to Acquiescing as equal to or less than the payoff for Escalating.

Call R_1 Player 1's resolve. Let s_1 be Player 1's payoff to acquiescing if Player 2 challenged. r is the probability that disaster will occur if a player escalates. Let w_1 be Player 1's prize if he escalates and no disaster occurs. Let d_1 be Player 1's payoff if he escalates and disaster occurs.

$$\begin{aligned} \text{U}_1(\text{Acquiesce}) &\leq \text{U}_1(\text{Escalate}) \\ s_1 &\leq w_1(1-r) + d_1 \cdot r \\ s_1 &\leq w_1 - w_1 r + d_1 \cdot r \\ r &\leq \frac{w_1 - s_1}{w_1 - d_1} = R_1 \end{aligned}$$

[1]Some material from this section is borrowed from Matt Levy and Josh Tasoff's 2007 notes.

We just found that whenever $r \leq R_1$ then Player 1 prefers to escalate. The only outcome allowed by a SPNE of the Brinkmanship game is for the player with the higher resolve to win. If the player with lower resolve goes first, he will simply choose not to confront. If the higher-resolve player goes first, he will confront and the other player will immediately acquiesce.

We can still solve this game through backwards induction. However, the use of the resolve value allows us to determine very quickly where in the game tree each player will no longer wish to escalate or challenge.

3.2 An example: 2 player game of brinkmanship

Take a look at the game tree labeled Figure 1.

First solve the game using backwards induction. What is the Nash equilibrium, the equilibrium path, and the equilibrium outcome (payoffs)?

Now let's calculate the resolve. What is each player's resolve in this game, and does it reinforce the equilibrium we just found using backwards induction?