Econ 110/PoliSci 135
Section 5 Notes

Anne Meng

October 1, 2013

1 Announcements

The second problem set is due this Thursday, Oct 3, at the beginning of lecture. Please note that there are two GSIs in this class with the same first name. Make sure that you turn in your problem set into my pile. Do not turn in your problem set to Anne Karing's pile (she is the other GSI). Also, please note that problem sets that recieve a score of 10 don't necessarily mean that there are no mistakes! Make sure to go through your problem sets after you get them back so that you don't make the same mistakes on the midterm.

2 Extensive Form Games

An extensive form (or dynamic) game requires us to define 5 things:

1. The players.
2. The order in which they make decisions (nodes).
3. What the players know when making decisions (information sets).
4. Players' rankings over outcomes (preferences/utility).
5. Probabilities of events outside the players' control (nature).

For now, we'll set aside the last item and only consider games of complete information. It won't be hard to add it back in later, but it will require a new solution concept to get any traction on that sort of game. Among the first four ingredients, the only thing that's new is that the order in which players make decisions now matters.

2.1 Setting up Dynamic Games

We often draw dynamic games in a tree form. There's no real trick to drawing extensive form games. Start with a node representing the first player (traditionally, leave this circle open to denote it as the *initial* node), and label it with the player's name. Then draw a branch for each of her actions, and label them with the actions' names. At the end of those, draw new nodes and label with the player who goes if player 1 takes each corresponding action. Repeat this until we reach *terminal* nodes, and then just put down the payoffs.

2.2 Information Sets

An **information set** is a collection of decision nodes that a player is incapable of distinguishing among.

Let's return to the congressional pay raise voting game. Players A, B, and C must decide whether to vote Yes or No to the pay raise proposed. How do we draw the following games, and how many information sets does each player have under each circumstance?

1. Everyone has full information.
2. No one can see what anyone else is voting for.
3. B can't see how A voted. C can't see how A voted; C can see how B voted.
4. B can see how A voted. C can see how A voted; C can't see how B voted.

After drawing out the various game trees, we can see the following:

1. A has 1 info set; B has 2 info sets; C has 4 info sets.
2. A has 1 info set; B has 1 info set; C has 1 info set.
3. A has 1 info set; B has 1 info set; C has 2 info sets.
4. A has 1 info set; B has 2 info sets; C has 2 info sets.

Note that everyone *always* only has 1 decision in this game, despite sometimes having multiple information sets, because each player is only deciding Yes or No once in this particular game.

2.3 Concepts Revisited

- **Strategy**: Recall that a strategy is a complete plan for playing the game. It must specify an action for each and every one of a player's information sets. This means that **players need to specify as many conditionalities as info sets in a strategy**.
- **Equilibrium Strategy Profile**: The list of equilibrium strategies for each player.
- **Equilibrium Path**: For an extensive form game, the sequence of actions induced by players using their equilibrium strategies.
- **Equilibrium Payoffs**: The payoffs at the end of the equilibrium path.

2.4 Backwards Induction

The method of **backwards induction** is the following process: start at the final decision node. Assume that the player will pick the action that maximizes her payoffs. Now consider the penultimate decision node. Go through the same process until you reach the initial decision note.

Remember that backwards induction can only be used for games of complete information. Also note that every game of perfect information with a finite number of nodes has a solution to backward induction. Moreover, if for for every player it is the case that no two payoffs are the same, then there is a unique solution to backwards induction.

2.4.1 An example

Let's return to the congressional pay raise voting game with complete information. If we solve the game using Backwards Induction, what is the Nash equilibrium?

- **Strategy**: Player A={N}, Player B={If Yes, then No; If No, then Yes}, Player C={If Yes, Yes, then No; If Yes, No, then Yes; If No, Yes, then Yes; If No, No, then No}
- **Equilibrium Strategy Profile**: {No; No, Yes; No, Yes, Yes, No} An acceptable shorthand is just to list all the strategies in order without writing the If-Then statements.
- **Equilibrium Path**: (No, Yes, Yes)
- **Equilibrium Payoffs**: (4,3,3)