Econ 110/PoliSci 135
Section 4 Notes

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1 Announcements

1.1 New Office Hours

New Office Hours for the rest of the semester: 2-4 PM on Tuesdays in 715 Barrows

2 Expected Utility

Given a lottery with outcomes $(X_1, X_2...X_n)$ with associated probabilities $(p_1, p_2...p_n)$ and a utility function u(x), we can calculate the following:

Expected Utility: EU $= p_1 * u(X_1) + p_2 * u(X_2) + \cdots + p_n * u(X_n)$
Certainty Equivalent: CE = X such that $u(X) = EU(lottery)$

2.1 An example

Suppose we have a lottery where you can win a prize of 100 dollars with probability p. We know that $u(0) = 0$ and $u(100) = 10$. We polled a student about her attitudes toward the lottery and ascertained the following data points:

p	**CE**
0.1	1
0.2	4
0.3	9
0.4	16
0.5	25

What would the graph of this student's VNM utility function look like? What is the utility function?

Now let's assume that this same student is offered a different lottery. If she enters this lottery, she will lose with p=0.5, win 5 dollars with p=0.30, win 10 dollars with p=0.10, win 15 dollars with p=0.05, or win 20 dollars with p=0.05.

What is the expected utility of this lottery? What would this student's certainty equivalent be?

3 Mixed Strategies

Once we convert the payoffs in a game into von Neumann-Morgenstern utility values, we can start to think about our players using mixed strategies. Players randomize between strategies at fixed universally known probabilities. Instead of each player declaring simultaneously their strategy, they declare the probability that they'll play each strategy.[1]

For a mixed-strategy profile to be a Nash Equilibrium, it must be the case that each player is indifferent between all their strategies. In other words, if Player 1 is choosing between strategies X and Y, then it must be the case that Player 1 is indifferent between strategies X and Y, keeping all other players' strategies fixed.

The reason for this is pretty clear. If one of the actions gave a higher expected utility, why would Player 1 want to play the other one? But this leads to an unusual state of affairs. If we're playing mixed strategies, the goal is not to maximize our own expected utility. The goal is to *make our opponent indifferent between the actions he's mixing over.*

Three key notes about mixed strategies: First, it is still perfectly acceptable to throw out any strictly dominated strategies – just as they were never part of a pure-strategy Nash equilibrium, they won't be in a mixed-strategy equilibrium. Second, *every* Finite Strategic Game has a Nash equilibrium (be it mixed or pure-strategy). Finally, if you find more than one pure strategy equilibrium in a game, there is also a mixed strategy equilibrium that blends them.

3.1 An example: The Coordination Game

		Campanile	**Sproul**
		α	1-α
Campanile	β	1,1	0,0
Sproul	1-β	0,0	2,5

Two student agree to meet for lunch on campus, but both of their phones have died. PANIC. They both independently decide that the two most likely places the other person is going to wait is by the Campanile or Sproul plaza. First observe that there are two pure-strategy Nash Equilibria. Now let's find the mixed strategy Nash equilibria.

Here's what to do when faced with this kind of question:

1. Determine what actions the players are mixing between. If a strategy is strictly dominated, you don't have to worry about it.
2. Assign probabilities to the strategies that each player will mix between.
3. Put yourself in P1's position, and make P2 indifferent between his strategies.
4. Put yourself in P2's position, and make P1 indifferent between his strategies.
5. Write down the equilibrium strategy profile.
6. Graph the best response functions.

[1]This section borrows from Matt Levy and Josh Tasoff's 2007 notes

In order for Player 1 to mix, he needs to be indifferent between C and S. Suppose Player 2 is playing $(\alpha C + (1-\alpha)S)$. What α will make player 1 indifferent between C and S?

P2 will play Campanile with probability α. We need to find what α is. To do so, we need to set $EU_1(C) = EU_1(S)$
Basically we need to make the EU of the rows (Campanile, Sproul) identical so that P1 will be indifferent between playing either strategy.

$$\begin{aligned} EU_1(C) &= \alpha \\ EU_1(S) &= 2(1-\alpha) \\ \Rightarrow \alpha &= 2(1-\alpha) \\ \boxed{\alpha = \tfrac{2}{3}} & \end{aligned}$$

Notice that it's the payoffs of Player 1 that determines Player 2's strategy. Now that Player 2 is set, let's suppose Player 1 is playing $(\beta C + (1-\beta)S)$. What β will make Player 2 indifferent between C and S?

$$\begin{aligned} EU_2(C) &= \beta \\ EU_2(S) &= 5(1-\beta) \\ \Rightarrow \beta &= 5(1-\beta) \\ \boxed{\beta = \tfrac{5}{6}} & \end{aligned}$$

The mixed strategy Nash Equilibrium of this game is therefore $(\frac{5}{6}C + \frac{1}{6}S, \frac{2}{3}C + \frac{1}{3}S)$.

Let's step back and go through our checklist:

1. No strategies are strictly dominated here, so we know that both players will be mixing between C and S.

2. P1 will play C with prob β and P2 will play C with prob α. It's our job to figure out what α and β are.

3. As P2, we set $EU_1(C) = EU_1(S)$ and determined that P1 will be indifferent between C and S when α is 2/3.

4. As P1, we set $EU_2(C) = EU_2(S)$ and determined that P2 will be indifferent between C and S when β is 5/6.

5. In equilibrium, the players will play the following strategies: (5/6(C) + 1/6(S), 2/3(C) + 1/3(S))

Finally, to graph the best response function, let's set the x-axis of the graph as α and the y-axis of the graph as β. Let's think about things from P1's perspective. P1 is going to be faced with α (because α is P2's strategy), but remember that P1 is going to be playing Campanile with probability β. When α=2/3, P1 is indifferent between C and S, therefore he will play $\beta \in [0, 1]$ because everything is a best response. When $\alpha > 2/3$, P1 will play β=1 (he will always go to the Campanile). When $\alpha < 2/3$, P1 will play β=0 (he will never go to the Campanile).

Go through the same process for P2. Where the two lines intersect is the MSNE for this game.

Hint: If you're having trouble with the graph, sometimes it's easier to draw the two best response graphs separately, and then combine them into one graph.

3.2 Another example

	L	C	R
T	6,1	3,0	1,-1
M	3,0	3,0	3,2
B	1,-1	5,2	6,0

Consider all pure strategies. Is anything strictly dominated for Player 1 or Player 2? What if we now consider mixed strategies?

Notice that no pure strategy is strictly dominated. However, if we're allowed to mix, then M is strictly dominated by playing (.5T+.5B).