Econ 110/PoliSci 135
Section 3 Notes

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1 Rent Seeking

Governments are powerful entities: they can make decisions affecting the behavior of firms or individuals. They can create valuable privileges by, for example, issuing licenses for certain trades. In response to this, we sometimes end up with a competition for these privileges - sometimes legal (lobbying), sometimes illegal (bribery). This is called **rent seeking**. When you have a monopoly, the excess profits are called **rent**.

How does the rent seeking game we went over in class relate to Normal Form Games we looked at last week? First of all, note that this is a Strategic Form Game because both players are making decisions simultaneously. Let's break down the components of the rent seeking game:

Strategy Set: M1 is Player 1's Strategy Set (also called a **choice variable**), and M2 is Player 2's Strategy Set.[1] The main difference between M and the strategy set in the Prisoner's Dilemma is that M is a *continuous choice variable*, whereas {Defect, Cooperate} are *discrete choice variables*. M can be any number from 0 to infinity - therefore we obviously don't want to draw out the matrix for this game.

A **utility** is a number assigned to an item that represents a measure of well-being that the item bestows upon a player. A **utility function** is a function that maps all the items to the utility assigned to each of them. An **expected utility** is the weighted average of the player's payoffs, where the weight assigned to a payoff is the probability of that payoff being received. In general, expected utility is relevant where there are *probabilities* associated with different outcomes.

We were told that both players have the following utility function:

$U_1 = (\frac{M1}{M1+M2})(V - M1) + (1 - \frac{M1}{M1+M2})(-M1)$

To find Player 1's best response function, we differentiated U_1 and set it to 0.
$\frac{dU_1}{dM1}=V[\frac{1}{M1+M2} - \frac{M1}{(M1+M2)^2}] - 1 = 0$

After we simplified everything, we ended up with the following best response function:
$M1 = \sqrt{VM2} - M2$

[1] We're assuming that M1 and M2 are both real numbers.

Likewise, if we follow the same procedure for Player 2, we end up with the following best response function:
$M2 = \sqrt{VM1} - M1$

Rewriting these two equations, we get the following:
$M1 + M2 = \sqrt{VM2}$ and $M2 + M1 = \sqrt{VM1}$

Therefore, we can conclude the following:
$\sqrt{VM2}$=$M1 + M2 = \sqrt{VM1}$
$\sqrt{VM2}$=$\sqrt{VM1}$
$M2 = M1$

Now that we know that $M2 = M1$ we can use this information to solve Player 1's best response function for M1.
$M1 = \sqrt{VM2} - M2$
$M1 = \sqrt{VM1} - M1$
$M1 + M1 = \sqrt{VM1}$
$2M1 = \sqrt{VM1}$
$(2M1)^2 = VM1$
$4M1^2 = VM1$
$4M1 = V$
$M1 = V/4$

By solving Player 1's best response function, we know what Player 1's equilibrium strategy will be: He will always play $M1 = V/4$. Note that by symmetry, we know that Player 2's equilibrium strategy will also be: $M2 = V/4$.

Moreover, by knowing what Player 1 and Player 2's strategies will be, we can calculate their probability of winning and their payoffs of playing the game. Recall that we were told that each player had the following utility function:
$U_1 = (\frac{M1}{M1+M2})(V - M1) + (1 - \frac{M1}{M1+M2})(-M1)$

Really, this utility function represents the Expected Utility from playing the game:
{the probability of winning times the payoff of winning} plus {the probability of losing times the payoff of losing}.

We know that the probability of winning is the following: $\frac{M1}{M1+M2}$
Since we know the equilibrium values of M1 and M2, we can plug in the values: $\frac{V/4}{V/4+V/4} = 1/2$

Now let's plug in the values for M1 and M2 into Player 1's utility function to see what his payoff is of playing the game:
$U_1 = (\frac{M1}{M1+M2})(V - M1) + (1 - \frac{M1}{M1+M2})(-M1)$
$U_1 = (1/2)(V - V/4) + (1 - 1/2)(-V/4)$
$U_1 = V/2 - V/8 - V/8$
$U_1 = V/4$

Each player will receive a payoff of $U_1 = V/4$ if he plays his equilibrium strategy $M1 = V/4$.

Note that it is just a coincidence that the equilibrium strategy and equilibrium payoff are the same in this problem! This is not always the case, and it is really important to remember to plug in the equilibrium strategy into the player's utility function to see what the payoff to playing the game is.

1.1 Taking a Step Back: Components of this Game

This is a **strategic form game** because both players move simultaneously in choosing what to set M. The **strategy set** of this game is M1 for Player 1 and M2 for Player 2. M can be any number the players choose. The **equilibrium strategy** for P1 is always play $M1 = V4$ - this is also called the **best response** to any strategy P2 can play. The **strategy profile** of the game is {P1 always plays M1=V/4; P2 always plays M2=V/4.} The **equilibrium payoff** for both players is (V/4, V/4). The strategy profile is a **Nash Equilibrium** because keeping the other player's strategy fixed, no player can deviate (aka: both players are playing a best response to each other).

2 Attitudes about Risk

Broadly speaking, **Von Newmann-Morgenstern (VNM) utility functions** are functions that map preferences to numbers and creates a simple way to compare choices. VNM utility functions take into account players' attitudes toward risk. *You can assume that the payoffs we see in this class are all VNM utilities.*

When considering games where there is uncertainty about the outcome (for example, lotteries), we need to look at Expected Utilities. Let's take a lottery with three outcomes: don't win (and get 0 dollars), win 5 dollars, win 10 dollars. If I participate in the lottery, the probability that I will win 0, 5, or 10 dollars are the following: $p_1 = .80, p_2 = .15, p_3 = .05$. What is the **expected value** of this gamble?

Expected Value: $EV = (.80)(0) + (.15)(5) + (.05)(10) = 1.25$

A **certainty equivalent** is the amount that a person would accept instead of playing the lottery. Attitudes toward risk are defined by a person's preferences between the Expected Value and Certainty Equivalent.

A person is **Risk Neutral** if he is indifferent between playing the lottery and the Certainty Equivalent: $CE = EU(lottery)$.
A person is **Risk Averse** if he prefers a Certainty Equivalent that is *lower* than the lottery itself rather than playing the lottery: $CE < EU(lottery)$.
A person is **Risk Acceptant** if he would only accept a Certainty Equivalent that is *higher* than the lottery. In other words, he loves playing the lottery so much, that you would actually have to pay him more to just take the money for sure: $CE > EU(lottery)$.

3 Expected Utility

Given a lottery with outcomes $(X_1, X_2...X_n)$ with associated probabilities $(p_1, p_2...p_n)$ and a utility function u(x), we can calculate the following:

Expected Utility: EU = $p_1 * u(X_1) + p_2 * u(X_2) + \cdots + p_n * u(X_n)$
Certainty Equivalent: CE = X such that $u(X) = EU(lottery)$