

# Econ 110/PoliSci 135

## Section 2 Notes

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September 10, 2013

### 1 Weakly Dominated Strategies

#### 1.1 Concepts

A strategy  $X$  is a **weakly dominated strategy** if there exists another strategy  $Y$  such that  $Y$  always does at least as good as  $X$  and sometimes better. Unlike **strictly dominated strategies**, where the strategy always had to do worse, indifference is now allowed when comparing payoffs.

Similar to IDSDS, we can now use iterated deletion of weakly dominated strategies (IDWDS) to try and solve normal form games. Note that the order of deletion matters if using Iterated Deletion of Weakly Dominated Strategies.

#### 1.2 An example

Solve the following normal form game by eliminating weakly dominated strategies:

	L	R
T	1, 1	0, 0
M	1, 1	2, 1
B	0, 0	2, 1

There are four possible solutions of this game: (T,L), (M,L), (R,M), and (B,R). The order in which you delete weakly dominated strategies govern which solution you end up at.

### 2 Nash Equilibrium

#### 2.1 Concepts

These are the main components of any game:

- **Strategy Set:** The collection of all feasible strategies for a player.
- **Strategy:** A fully specified decision rule for how to play a game that accounts for *every contingency*. A strategy should specify what to do in every situation (even if you don't think that the situation will play itself out).
- **Strategy Profile:** A collection of strategies, one for each player.

- **Best Response:** A strategy that maximizes a player's payoff, given his beliefs about what strategy the other player will play.

A **Nash Equilibrium** is a strategy profile such that keeping all other players' strategies fixed, no player can profitably deviate. Another way to think about this is that each player is playing a best response to what the others are playing.

In checking to see if a strategy profile is a Nash, you should ask the following questions: (for simplicity, assume that this is a 2 person Prisoner's Dilemma game, and you want to check if (Defect, Defect) is a Nash)

- Is Defect a best response for Player 1 if Player 2 is playing Defect? In other words, given that Player 2 is going to play Defect, can Player 1 profitably deviate to Cooperate?
- Similarly, is Defect a best response for Player 2 if Player 1 is playing Defect?

## 2.2 Applying these concepts to the Prisoner's Dilemma

	<b>D</b>	<b>C</b>
<b>D</b>	2, 2	4, 1
<b>C</b>	1, 4	3, 3

- Payoffs: 1,2,3,4
- Outcomes: (D,D), (C,C), (D,C), (C,D)
- Strategy Set: Cooperate, Defect
- Strategy: Player 1 always play Defect.
- Strategy Profile: Player 1 always play Defect. Player 2 always play Defect.
- Best Response: If P2 plays D, then P1's best response is D. If P2 plays C, then P1's best response is D. Same for P2.
- Nash Equilibrium: (Defect, Defect)

## 2.3 Finding Nash Equilibria

There is an easy mechanical way to find Nash Equilibria in normal form games. Start with Player 1, and go through each possible strategy Player 2 can play (ie: the columns). For each strategy (column), underline Player 1's highest payoff(s). If two payoffs are tied, underline them both. After you're finished with Player 1, repeat the same process for Player 2. After you're done underlining for both players, the cells where *both* payoffs are underlined are Nash. Note that it is possible to have multiple Nash equilibria.

Also note that when we are underlining the highest payoff(s), we are simply finding each player's best response for every action their opponent can take. A solution is Nash if every player is playing a best response to everyone else.

## 2.4 An Example

	A	B	C	D	E
V	0,0	3,2	1,7	6,4	2,5
W	1,2	1,1	0,4	0,3	0,1
X	4,6	2,2	1,4	2,5	1,3
Y	4,6	2,2	4,4	1,6	5,4
Z	2,1	3,2	1,5	2,1	3,2

Are any strategies strictly dominated? Weakly dominated? Find the Nash equilibria of the game using the underlining method. List each component of the game (What is the strategy set? Strategies? etc).

## 2.5 Cournot Competition

Two firms simultaneously decide how much to produce. Each firm has the option of producing 2, 3, or 4. The cost of producing each unit is 1. The price for each unit is given by  $10 - (q_1 + q_2)$ .

1. Specify the normal form of this game.
2. Try to solve this normal form game by IDSDS, IDWDS, and underlining.

## 3 Taking a Step Back: Relating Solution Concepts

So far we have learned 3 types of solution concepts:

- **strict dominance**
- **weak dominance**
- **Nash Equilibrium**

Weak dominance requires a stronger assumption than strict dominance because we are assuming that players will not play a strategy that is *as good as* but worse off in at least one instance than another strategy. Contrast this with strict dominance where we are only assuming that players will not play a strategy that is always worse off.

What is the relationship between strict/weak dominance and Nash? IDSDS will not eliminate any Nash Equilibria, but IDSWWS can. Any solution generated through IDSDS or IDSWWS will be a Nash equilibrium. However, not every Nash can be obtained through IDSDS or IDSWWS.

*The main tradeoff to keep in mind* is that as we move from strict dominance to weak dominance to Nash, stronger assumptions are necessary, but the solution concepts become more useful in narrowing down the set of predicted outcomes.