

## MATH CAMP

## Problem Set 1

1. S&B 3.9 a, c, e, f, and g, p. 57
2. The utility that a legislator obtains from a policy in a one-dimensional policy space is  $U(x) = -a(x - i)^2$  where  $i$  is the legislator's ideal point and  $a$  is the salience of the issue. This utility function formalizes the idea that the farther the policy  $x$  is from the legislator's ideal policy  $i$ , the worse off the legislator is. As we will see later in the course, the larger  $a$ , the larger the effect that a given change in  $x$  has the legislator's utility, i.e., the issue is more salient. To make things more concrete, suppose  $a = 2$  and  $i = 60$ .
  - (a) Prove that the legislator's utility maximizing policy is  $x = 60$ .
  - (b) Suppose that the constitution requires  $x$  to be in the range  $x \in [70, 80]$ . What policy in this range maximizes the legislator's utility. Be sure to justify your answer.
3. The balance-of-power school in international relations argues that peace is most likely when there is a balance of power between states. By contrast, the preponderance-of-power school argues that that peace is *least* likely when there is a balance of power and is most likely when one side has a preponderance of power. Suppose that the analysis of a formal model yields the result that the probability of war in the model is

$$\pi = \frac{1}{4} - \left(p - \frac{1}{2}\right)^2$$

where  $\pi$  is the probability of war and  $p$  is the distribution of power between state  $s$  1 and 2. That is,  $p$  is the probability that 1 prevails in a war against 2. There is a balance of power when  $p = 1/2$ . By contrast, 1 has a preponderance of power (in the extreme) when  $p = 1$  and 2 has a preponderance of power when  $p = 0$ . Is the result of the formal analysis consistent with one of the two schools or neither? Be sure to explain your answer.