PS230: ESSENTIAL MATHEMATICAL TOOLS

Final Exam, Fall 2011

Answer all of the following questions. Except for problem 9 and 10 (b), each question or part of a question is worth 10 points (e.g., part (a) of a question is worth 10 points, part (b) is worth 10 points, and so on). Problem 9 is worth 20 points; 10 (b) is worth 40.

1. Let $y = 3x^4 + 8x^3 - 18x^2 + 12$.

   (a) Show that x = -3, x = 0, and x =1 are critical points of y.

   (b) Which of these critical points are local minima and maxima? Be sure to justify your answer.

   (c) What are the global maxima and minima over the interval $x \in [-1, 2]$?

2. Find dz / dx by implicit differentiation of $(x+z)^3 = xz(x+1)$.

3. Calculate the indefinite integral $\int \frac{2e^{4(2x-3)^{-2}}}{(4x-6)^3} dx$.

4. The random variable y is distributed according to the density function:

$$d(y) = \begin{cases} 0 & \text{if } y < -1 \\ sy^2 & \text{if } -1 \le y \le 1 \\ 0 & \text{if } 1 < y \end{cases}$$

   where s is a parameter to be determined.

   (a) What is the value of s?

   (b) What is the probability that y is between ½ and 1?

   (c) What is the expected value of y?

   (d) What is the variance of y?

   (e) What is the cumulative distribution function $D(y)$ for any $y \in [-1,1]$?

5. A gambler is unsure if a coin is fair or biased and thinks that the probability that it is biased is 1/3. If the coin is fair, the probability of a heads on a flip is ½. If the coin is biased, the probability of a heads is 3/8. The coin is flipped twice and comes up tails both

times. In light of these observations, what is the gambler's posterior probability that the coin is fair?

6. Let

$$A = \begin{bmatrix} 2 & -2 & -1 \\ 0 & 1 & 3 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & -2 & 3 \\ 1 & 1 & -3 \end{bmatrix}$$

(a) Find $A + B$.

(b) Find B^T.

(c) Find AB^T.

7. Let $\mathbf{u} = (1, -4, 3, -1)$ and $\mathbf{v} = (2, 0, 1, 2)$.

(a) What is $\mathbf{u} + \mathbf{v}$?

(b) What is the cosine of the angle between $\mathbf{u}$ and $\mathbf{v}$?

(c) What is the vector whose norm is twice that of $\mathbf{u}$ and which points in the opposite direction of $\mathbf{u}$?

8. Let $h(x, y) = \sqrt{\ln xy}$. Find $\partial h / \partial x$ and $\partial^2 h / \partial y \partial x$.

9. (20 pts) Let $g(x, y) = -x^2 - 3xy - 5y^2 + 26x + 72y - 268$. Find the critical points of *g*.

10. Continue to assume $g(x, y) = -x^2 - 3xy - 5y^2 + 26x + 72y - 268$. Suppose we want to find the maximum of *g* subject to the constraint $y \leq 8 - 4x$.

(a) What is the Lagrangian for this problem?

(b) (40 pts) What are the five conditions that the solution to the Lagrangian must satisfy.

(c) Suppose that when you find a solution to the conditions in (b), the multiplier is positive. What if anything does this tell you?