

PS230: ESSENTIAL MATHEMATICAL TOOLS

Final Exam, Fall 2008

Answer all of the following questions. Each question or part of a question is worth 10 points (e.g., part (a) of a question is worth 10 points, part (b) is worth 10 points, and so on).

1. Find the derivative of $y = x^3 \ln\left(\frac{x^2}{e^{3x}}\right)$.
2. Let $z(x) = x^3 - 3x^2 + 9$.
 - (a) What are the critical points of z ?
 - (b) What are the local minima and maxima? Be sure to justify your answer.
 - (c) What are the global maxima and minima over the interval $x \in [-1, 2]$?
3. Find dy/dx by implicit differentiation of $(yx^2)^3 = y^2 - 2$.
4. Let $f(x) = \frac{1}{(2x+1)^3}$.
 - (a) Verify that $F(x) = -\frac{1}{4(2x+1)^2}$ is the anti-derivative of f .
 - (b) What is the area under f between 1 and 2?
4. Calculate the indefinite integral $\int \frac{3y \ln(3y^2 + 1)^2}{3y^2 + 1} dy$.
5. Suppose that a coin is slightly unfair. The probability of heads is .6 and the probability of tails is .4.
 - (a) What is the probability of getting 4 heads out of 6 flips? In writing out the answer, you can leave any number raised to an exponent in that form.
 - (b) What is the probability of getting 4 heads out 6 flips **or** 2 tails out of 6 flips? Be sure to justify your answer. Again, you can leave any number raised to an exponent in that form.

6. The random variable n is distributed over the non-negative integers $0, 1, 2, 3, \dots$ according to the cumulative distribution function $F(n) = c - e^{-2n}$.

- (a) What is the value of c ?
- (b) What must the value of $\lim_{n \rightarrow \infty} F(n)$ be?

7. Let

$$A = \begin{bmatrix} 1 & 3 \\ -2 & 2 \\ 5 & -1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix}$$

- (a) Find AB .
- (b) Does $AB=BA$? Show that it does or explain why it does not.
- (c) Find B^{-1} .

8. Let $\mathbf{r} = (3, -2, 2)$ and $\mathbf{s} = (1, -1, 2)$.

- (a) What is $\mathbf{r} \cdot \mathbf{s}$?
- (b) What is $\|\mathbf{r} - \mathbf{s}\|$?
- (c) Is the angle between \mathbf{r} and \mathbf{s} less than, equal to, or greater than 90 degrees?

11. Let $h(x, y) = (x - y^2)^{2/3}$. Find $\partial^2 h / \partial x \partial y$.

12. In Dal Bó and Powell's model of civil conflict (*AJPS* 2008) the probability of war π is given by $\pi = f(c^{p(1-\delta)/\delta})$ where f is a strictly increasing function; c is the level of national income; $p \in [0, 1]$ is the strength of the out-of-power faction; and $\delta \in [0, 1]$ measures the cost of fighting (and higher δ is a higher cost). Does an increase in the cost of fighting δ make war more or less likely?