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PS230: MATH FOR POLITICAL SCIENTISTS

Final Exam, Fall 2007

Answer all of the following questions. Each question or part of a question is worth 10 points (e.g., part (a) of a question is worth 10 points, part (b) is worth 10 points, and so on).

1. Find the derivative of $y = \ln x^3 + 6x^2e^{2x}$.

2. Find dy/dx by implicit differentiation of $x^2y^2 = 2x^2 + y^2$

3. Suppose $y = h(x)$, $dy/dx = 0$ at $x = 5$, and $d^2y/dx^2 < 0$ at $x = 5$. What do we know about $h(5)$?

4. Calculate the indefinite integral $\int 6(1+z^{-3})e^{2z-z^{-2}+3}dz$.

5. The random variable x can take on the values -2, 0, and 2 with probabilities 1/6, 1/6, and 4/6 respectively. What is the expected value of x? What is the variance of x (write an expression for calculating the variance but you do not need to do the arithmetic of actually calculating it)?

6. The random variable z is distributed according to the density function:

$$f(z) = \begin{cases} 0 & \text{if } z < -1 \\ h(1-z^2) & \text{if } -1 \le z \le 1 \\ 0 & \text{if } 1 < z \end{cases}$$

where h is a parameter to be determined.

   (a) What is the value of h?

   (b) Find the cummulative distribution function $F(z)$ for any z is between -1 and 1. (If you were unable to answer (a), find the cummulative distribution as an expression involving h.)

   (c) What is the probability that z is between 0 and 1/2? (If you were unable to answer (a), find the cummulative distribution as an expression involving h.)

7. Suppose the probability of observation O_1 given hypothesis H_1 is $P(O_1 \mid H_1) = 1/3$, the probability of observation O_2 given hypothesis H_1 is $P(O_2 \mid H_1) = 2/3$, the probability of observation O_1 given hypothesis H_2 is $P(O_1 \mid H_2) = 2/3$, and the probability of observation O_2 given hypothesis H_2 is $P(O_2 \mid H_2) = 1/3$. The prior probabilities of hypotheses H_1 and H_2 are $P(H_1) = 1/2$ and $P(H_2) = 1/2$. Research then yields observation O_1. Given this observation, what is the probability of H_1, i.e., what is $P(H_1 \mid O_1)$?

8. Is *A* a left inverse of *B* where

$$A = \begin{pmatrix} 1 & -1 & 2 \\ 2 & 1 & -1 \\ 3 & 2 & -2 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 0 & 2 & -1 \\ 1 & -8 & 5 \\ 1 & -5 & 3 \end{pmatrix}$$

9. Consider the non-square matrix $M^{s \times n}$ and suppose that the matrix $R^{n \times n}$ is a right inverse of $M^{s \times n}$. Can $R^{n \times n}$ be a left inverse of $M^{s \times n}$? If not, explain why not. If it can be a left inverse under some circumstances, explain what those circumstances are.

10. Let $\mathbf{u} = (5, -2, 1)$, $\mathbf{w} = (-2, -2, -6)$, and $\mathbf{v} = (-4, 10)$. Are the vectors **u** and **w** perpendicular? Are **u** and **v** perpendicular?

11. Let $f(x, y) = e^{xy}$. Find $\partial f / \partial x$ and $\partial^2 f / \partial y \partial x$.

12. Let $g(x, y) = \ln\left(\frac{x}{y}\right)$. Find $\partial g / \partial x$ and $\partial^2 g / \partial y \partial x$

13. Acemoglu and Robinson (2006) derive a nonmontonic relationship between democratization and income inequality. Authoritarian states are unlikely to democratize at low levels of inequality, are more likely to democratize at moderate levels of inequality, and are again unlikely to democratize (and are very likely to engage is extensive repression) at high levels of inequality.

    Suppose that the probability that an authoritarian state democratizes, δ, is given by:

$$\delta = \left(\frac{2}{w} - \sigma\right)\left(\frac{\sigma}{r} - 4\right)$$

    where $\sigma > 0$ is the level of inequality, $w > 0$ is the level of national income, and $r > 0$ is the cost of repression.

    We are interested in seeing how δ varies as σ does while holding *w* and *r* constant.

(a) Find the σ at which δ attains a maximum? Hint: Solve $\partial\delta / \partial x = 0$ for σ and then show that this maximizes δ for fixed *w* and *r*.

(b) Let σ^* be the solution found in (a). Show that the probability of democratization increases as inequality rises for levels of σ below σ^* and that the probability of democratization decreases as inequality rises for levels of σ above σ^*.

14. Let $h(x, y) = (x-1)^4 + 2(x-1)^2(y-2)^2 + (y-2)^4$.

(a) Finding the critical point(s) of *h*.

(b) Can you say whether the critical point(s) is/are a local minima, local maxima, global minima, or global maxima?