Match Camp Diagnostic Exam, August 2015

You have two hours to complete the exam. Please show your work and ***circle or box your answers.***

1. Suppose that a state legislator, Frank, is trying to decide whether to enter a race for US Congress. The seat is currently held by an incumbent from the other party, and Frank is trying to guess whether the incumbent is going to run for re-election. Suppose that Frank's prior belief about the probability that the incumbent *will* run for reelection is 0.70. Frank also believes that the probability that the incumbent will hold a large fundraising pool party this month is 0.6 if she *is* running for reelection and 0.2 if she is not. In other words, $Pr(poolparty|reelection) = 0.6$ and $Pr(poolparty|noreelection) = 0.2$. Let's say Frank *does not* observe a large fundraising pool party this month. Apply Bayes' rule to figure out Franks updated beliefs about the probability the incumbent will run for reelection, conditional upon this new evidence.

2. The random variable y is distributed according to the following density function:

$$g(y) = \begin{cases} 0, & \text{if } y < 1 \\ -\frac{1}{p}(1+y), & \text{if } 1 \leq y \leq 5 \\ 0, & \text{if } 5 < y \end{cases}$$

   Where p is a parameter to be determined.

   (a) Are you looking at a PDF or CDF? Is it discrete or continuous?

   (b) What is p?

   (c) What is the PDF of g(y) $\forall y \in [1, 5]$?

   (d) What is the CDF of G(y) $\forall y \in [1, 5]$?

   (e) What is the probability that y is between 2 and 3?

   (f) What is the expectation of y?

3. Suppose that the random variables x and y can take on the values -2 and 4, and the random variables are jointed distributed as follows:

		y	
x		**-2**	**4**
	-2	0	1/4
	4	1/8	5/8

   (a) What is the mean of X and the mean of Y?

(b) What is the variance of X and the variance of Y?

(c) What is the covariance of X and Y?

(d) What is the conditional mean of Y, given that x=4?

4. Let $h(x, y, z) = 2x^2y + (zx - y^4)(y + 2)$. What is $\frac{\partial h}{\partial y}$? What is $\frac{\partial^2 h}{\partial y \partial x}$?

5. Let $f(x) = \frac{x^2}{e^x}$ on the domain of $x \in [0, 10]$. Find all the critical points, state whether it is a max or a min (and justify your answer), then state whether it is a global max or min along this domain.

6. A multivariate regression model looks like $\mathbf{y}= \beta_1 x_1 + \beta 2_{x2} + ... + \beta_n x_n + \epsilon$. With only one data point, each of the $\mathbf{y}$, x_j, and ϵ would be scalar, but with many data points, they are each column vectors. With β a a column vector and X a matrix containing each x_j in a different column, we can write the model simply as $\mathbf{y}=X\beta+\epsilon$. Minimizing the squared error amounts to minimizing $\epsilon^*\epsilon$, which results in $\beta=(X^TX)^{-1}X^Ty$.

   If your independent variable is $\mathbf{y}= \begin{bmatrix} 7 \\ 2 \end{bmatrix}$ and your two dependent variables are $x_1= \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $x_2= \begin{bmatrix} 3 \\ 2 \end{bmatrix}$, what is your estimate of β?

7. Compute the directional derivative of $f(x, y) = 4x^2 + y^2$ at the point (1,1) in the direction of $(\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}})$.

8. Suppose Anne's utility function regarding a three dimensional policy space (x,y,z) is given by $U(x, y, z) = xyz + z$ subject to the following constraints $x^2 + y^2 + z \leq 6, x \geq 0, y \geq 0, z \geq 0$.

   (a) Set up the first order conditions for the *unconstrained* maximization problem. You do *not have* to solve for the critical points, just set the conditions up.

   (b) Now consider the constrained problem. Please set up the Lagrangian function and the Kuhn-Tucker conditions. You do not have to solve for the critical points.

   (c) Suppose that only the Lagrangian multiplier for the second, third, and fourth constraints turn out to the positive. What does that tell you?

That's it! :D