August 25, 2014

MATH CAMP FINAL AND EQUIVALENCE EXAM

You have two hours to complete the exam. Answer all of the following questions and show your work. Each question or part of a question is worth 10 points (e.g., part (a) of a question is worth 10 points, part (b) is worth 10 points, and so on). There are 160 possible points. Passing the exam requires a score of 120 or more. **Please circle or box your answers.** Good luck.

1. Suppose $h(x,y) = e^{2x+3yx^2}$ What is $\partial^2 h / \partial y \partial x$?

2. An insurance company has three types of policyholders: high risk (20% of policyholders), medium risk (30%), and low risk (50%). The probability that a customer has at leaSt one accident in a given time period is 25% for high risk, 16% for medium risk, and 10% for low risk.

   (a) What is the probability that a policyholder chosen at random will have at least one accident in the current time period?

   (b) What is the probability that a policyholder is high risk, given that the person had at least one accident in the current time period.

3. The random variables $r \in \{-2, 2\}$ and $s \in \{-1, 0, 1\}$ are jointly distributed according to

s \ r	-2	2
-1	0	1/3
0	1/6	1/6
1	1/3	0

   (a) What is the mean of *r*?

   (b) What is the variance of *s*?

   (c) What is the covariance between *r* and *s*?

   (d) What is the conditional mean of *s* given *r* = 2?

4. Show that $\text{cov}(VW) = E(VW) - E(V)E(W)$.

5. Let $G = \begin{bmatrix} 3 & -2 & 0 \\ 1 & -1 & -2 \end{bmatrix}$ and $H = \begin{bmatrix} 2 & -2 & 0 \\ 1 & 0 & 1 \end{bmatrix}$. Find GH^T.

6. What is the cosine of the angle between the vectors **s** = (3, 0, -4) and **x** = (2, -1, -2)?

7. Let $g(z) = \dfrac{e^{z^3-8}}{z+1}$. Is $z = 2$ a critical point of g?

8. Let $4xy = (3y - x)^2(2x - y)$. Find dy/dx.

9. Let $f(x, y) = xy^2 + x^3y$.

   (a) What is the rate of change of f at (2,-1) in the direction of $(1/\sqrt{10}, 3/\sqrt{10})$?

   (b) In what direction is f increasing most rapidly at (2,-1)?

10. A voter's ideal point in the two-dimensional policy space (e,s) is (3,4) and her utility is given by $W(e,s) = -[2(e-3)^2 + (s-4)^2]$. She is trying to maximize her utility subject to $s \geq 1$, $e \geq 1$, and $es \leq 4$.

    (a) What is the Lagrangian for this optimization problem?

    (b) Suppose that the Lagrangian mulitplier associated with the first constraint turns out to zero and that associated with the third constrain is postive. What does this tell you?