August 23, 2013

MATH CAMP FINAL AND EQUIVALENCE EXAM

You have two hours to complete the exam. Answer all of the following questions and show your work. Each question or part of a question is worth 10 points (e.g., part (a) of a question is worth 10 points, part (b) is worth 10 points, and so on). There are 200 possible points. Passing the exam requires a score of 150 or more. Good luck.

1. Suppose $x^3y + 2 = 3(x+y)(x-1)^2$. What is *dy/dx*?

2. Each of the random variables *y* and *z* can take on the values 1 or -1, and the variables are jointely distributed according to

		z = -1	*z* = 1
y	-1	1/8	3/8
	1	3/8	1/8

   (a) What are the expected values of *y* and *z*?

   (b) What are the variances?

   (c) What is the covariance between *y* and *z* and the correlation between them?

3. The cummulative distribution function for the random variable *w*

$$G(w) = \begin{cases} 0 & \text{if } w < 0 \\ -kw(w-4) & \text{if } 0 \le w \le 2 \\ 1 & \text{if } 2 < w \end{cases}$$

   where *k* is a parameter to be determined.

   (a) What is the value of *k*?

   (b) What is the probability that *w* is between 0 and 1?

   (c) What is the expected value of *w*?

(d) What is the variance of *w*?

(e) What is the density of $G(w)$ for any $w \in [0,2]$?

4. Show that $V(x+y) = V(x) + V(y) + 2\text{cov}(x,y)$. Hint: $V(x+y) = E([x+y-E(x+y)]^2)$.

5. Let

$$F = \begin{bmatrix} 1 & 0 \\ 4 & -3 \\ 2 & 1 \end{bmatrix} \text{ and } G = \begin{bmatrix} 1 & -2 & 1 \\ 1 & 0 & -2 \end{bmatrix}$$

(a) Find *GF*.

(b) Find $(FG)^T$.

(c) Find $(GF)^{-1}$.

6. Let $v(r,s) = e^{\sqrt{rs}}$.

(a) Find the gradient ∇v.

(b) In what direction is *v* increasing most rapidly starting from $r = s = 1$?

7. Let $h(x,y) = 5 - (y-1)^2(x-3)^6$.

(a) Find $\partial h / \partial x$.

(b) Find $\partial h / \partial y$

(c) Find the critical point or points of *h*.

8. A decision-maker is chosing how much time to spend on activities *x* and *y*. Her utility is given by $U(x,y) = x(y-2)^3$. Her levels of effort are also constrained by two conditions, $y \le 8 - x^3$ and $12 \ge y + 3x$.

(a) What is the Lagrangian for this optimization problem?

(b) Suppose that the Lagrangian mulitplier associated with the first constraint turns out to be postive. What does this tell you?